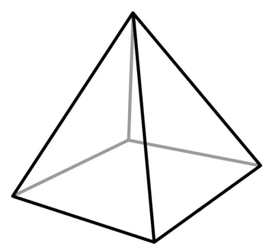
Exam Symmetry in Physics

Date	May 8, 2014
Room	A. Jacobshal 01
Time	18:30 - 21:30
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the 3 exercises (18 in total) have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

Consider a pyramid with a regular square as base and its apex above the center of the square (see figure). Its symmetry group is C_{4v}.



(a) Identify all transformations that leave this pyramid invariant.

(b) Show that C_{4v} is isomorphic to D_4, for instance using cycle notation.

(c) Argue, using geometrical arguments, that C_{4v} has five conjugacy classes.

(d) Construct the character table of C_{4v}.

(e) Construct explicitly the three-dimensional vector representation D^V for the two transformations that generate C_{4v}.

(f) Decompose D^V of C_{4v} into irreps and use this to conclude whether this group allows in principle for an invariant three-dimensional vector, such as an electric dipole moment.

(g) Determine the Clebsch-Gordan series of the direct product representation $D^V \otimes D^V$ of C_{4v} and give the number of independent invariant tensors (if any).

Exercise 2

Consider a classical potential $V(|\vec{r}|)$ experienced by a charged particle at position $\vec{r} = (x, y, z)$ in three dimensions.

(a) Show that the potential V is invariant under $O(3)$ transformations.

(b) What is the defining representation of $O(3)$?

(c) Explain the difference between the vector and axial-vector representations of $O(3)$.

(d) Show that the Kronecker delta δ_{ij} is invariant under $O(3)$ transformations.

(e) Determine the subgroup of $O(3)$ transformations that leave the tensor $\sigma_{ij} = \delta_{ij} + c\delta_{i1}\delta_{j1}$ invariant, for a nonzero constant c.

(f) Explain what is a pseudoscalar and give an example of one.

Exercise 3

(a) Show that the group elements of $O(2)$ have either determinant 1 or -1.

(b) Show that the elements of $O(2)$ with determinant 1 form a group, whereas the elements with determinant -1 do not.

(c) Show that $SO(2) \cong U(1)$.

(d) Write down a three-dimensional rep of $SO(2)$ and explain whether it is an irrep or not.

(e) Give an example of a physical system with an $SO(2)$ or $U(1)$ symmetry.